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New physics effects in the rare $B_s \to \gamma\,\ell^+\ell^-$ decays with a polarized photon

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Abstract. Using the most general model independent form of the effective Hamiltonian, the rare $B_s \to \gamma\,\ell^+\ell^-$ decays are studied by taking into account the polarization of the photon. The total and the differential branching ratios for these decays, when the photon is in the positive or the negative helicity state, are presented. The dependence of these observables on the new Wilson coefficients is studied. Also is investigated the sensitivity of the "photon polarization asymmetry" in $B_s \to \gamma\,\ell^+\ell^-$ decays to the new Wilson coefficients. It has been shown that all these physical observables are very sensitive to the existence of new physics beyond SM and their experimental measurements can give valuable information on it.

1 Introduction

The rare B-meson decays induced by the flavor-changing neutral currents have always been important channels for obtaining information on the fundamental parameters of the standard model (SM), testing its predictions at loop level and probing possible new physics.

The observation of radiative penguin mediated processes, in both the exclusive $B \to K^*\gamma$ [2] and inclusive $B \to X_s\gamma$ [3] channels have prompted the investigation of the radiative rare B-meson decays with a new momentum. Among these, the semileptonic $B_s \to \gamma\,\ell^+\ell^-$ ($\ell = e, \mu, \tau$) decays have received special interest due to their relative cleanliness and sensitivity to new physics as well as ongoing experiments at the two B-factories [4, 5]. It is well known that the corresponding pure leptonic processes $B_s \to \ell^+\ell^-$ have helicity suppression so that their decay widths are too small to be measured for the light lepton modes. In SM the branching ratios are $\mathrm{BR}(B_s \to e^+e^-,\ \mu^+\mu^-) \simeq 4.2 \times 10^{-14}$ and 1.8×10^{-9}, respectively. Although the τ channel is free from this suppression, its experimental detection is quite hard due to the low efficiency. In $B_s \to \tau^+\tau^-\gamma$ decay, helicity suppression is overcome by the photon emission in addition to the lepton pair. Therefore, one expects for $B_s \to \gamma\ell^+\ell^-$ decay a larger branching ratio and this makes its investigation interesting. Indeed, $B_s \to \gamma\ell^+\ell^-$ decays have been widely investigated in the framework of the SM for light and heavy lepton modes [6–9], and one reported $\mathrm{BR}(B_s \to \gamma\,e^+e^-,\ \gamma\,\mu^+\mu^-,\ \gamma\,\tau^+\tau^-) = 2.35 \times 10^{-9}$, 1.9×10^{-9} and 9.54×10^{-9}, respectively. The new physics effects in these decays have also been studied in some models, like the minimal supersymmetric standard model (MSSM) [10–12] and the two Higgs doublet model [14–17], and it was shown that different observables, like the branching ratio, forward–backward asymmetry, etc., are very sensitive to the physics beyond the SM. An investigation of the polarization effects may provide another efficient way to establish the new physics. Along this line, the polarization asymmetries of the final state lepton in $B_s \to \gamma\ell^+\ell^-$ decays have been studied in MSSM in [13], and it was concluded that they can be very useful for an accurate determination of the various Wilson coefficients.

In a radiative decay mode like ours, the final state photon can also emerge with a definite polarization and provide another kinematical variable to study the new physics effects [11]. In this paper, we will study the rare $B_s \to \gamma\,\ell^+\ell^-$ decay by taking into account the photon polarization. Although an experimental measurement of this variable would be much more difficult than that of e.g., the polarization of the final leptons in $B_s \to \gamma\,\ell^+\ell^-$ decay, this is still another kinematical variable for studying radiative decays. In our work we will investigate the sensitivity of such a "photon polarization asymmetry" in $B_s \to \gamma\,\ell^+\ell^-$ decay to the new Wilson coefficients, in addition to the study of the total and differential branching ratios with a polarized final state photon. Doing this we use a most general model independent effective Hamiltonian, which contains the scalar and tensor type interactions as well as the vector types (see (1) below). We note that in a recent work [18] we have studied the related mode $B_s \to \gamma\,\nu\bar{\nu}$ with a polarized photon in a similar way and showed that the spectrum is sensitive to the types of the interactions so that it is useful to discriminate the various new physics effects.

This paper is organized as follows. In Sect. 2, we present the most general, model independent form of the effective Hamiltonian and the parameterization of the hadronic matrix elements in terms of the appropriate form factors. We then calculate the differential decay width and the photon polarization asymmetry for the $B \to \gamma\,\ell^+\ell^-$ decay when the photon is in the positive and negative helicity states. Section 3 is devoted to a numerical analysis and a discussion of our results.

2 Matrix element for the $B_s \to \gamma\,\ell^+\ell^-$ decay

The matrix element for the process $B_s \to \gamma\,\ell^+\ell^-$ can be obtained from that of the purely leptonic $B \to \ell^+\ell^-$ decay. Therefore, we start with the effective Hamiltonian for the $b \to s\ell^+\ell^-$ transition written in terms of twelve model independent four-Fermi interactions as follows [19]:

$$\begin{aligned}
\mathcal{H}_{\rm eff} &= \frac{G\alpha}{\sqrt{2}\pi} V_{ts}V_{tb}^* \\
&\times \Bigg\{ C_{\rm SL}\, \bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}\,L\,b\,\bar{\ell}\gamma^\mu\ell + C_{\rm BR}\,\bar{s}\mathrm{i}\sigma_{\mu\nu}\frac{q^\nu}{q^2}\,R\,b\,\bar{\ell}\gamma^\mu\ell \\
&+ C_{\rm LL}^{\rm tot}\,\bar{s}_{\rm L}\gamma_\mu b_{\rm L}\,\bar{\ell}_{\rm L}\gamma^\mu\ell_{\rm L} + C_{\rm LR}^{\rm tot}\,\bar{s}_{\rm L}\gamma_\mu b_{\rm L}\,\bar{\ell}_{\rm R}\gamma^\mu\ell_{\rm R} \\
&+ C_{\rm RL}\,\bar{s}_{\rm R}\gamma_\mu b_{\rm R}\,\bar{\ell}_{\rm L}\gamma^\mu\ell_{\rm L} + C_{\rm RR}\,\bar{s}_{\rm R}\gamma_\mu b_{\rm R}\,\bar{\ell}_{\rm R}\gamma^\mu\ell_{\rm R} \\
&+ C_{\rm LRLR}\,\bar{s}_{\rm L}b_{\rm R}\,\bar{\ell}_{\rm L}\ell_{\rm R} + C_{\rm RLLR}\,\bar{s}_{\rm R}b_{\rm L}\,\bar{\ell}_{\rm L}\ell_{\rm R} \qquad (1)\\
&+ C_{\rm LRRL}\,\bar{s}_{\rm L}b_{\rm R}\,\bar{\ell}_{\rm R}\ell_{\rm L} + C_{\rm RLRL}\,\bar{s}_{\rm R}b_{\rm L}\,\bar{\ell}_{\rm R}\ell_{\rm L} \\
&+ C_{\rm T}\,\bar{s}\sigma_{\mu\nu}b\,\bar{\ell}\sigma^{\mu\nu}\ell + \mathrm{i}C_{\rm TE}\,\epsilon^{\mu\nu\alpha\beta}\bar{s}\sigma_{\mu\nu}b\,\bar{\ell}\sigma_{\alpha\beta}\ell \Bigg\}\,,
\end{aligned}$$

where L and R are the chiral projection operators defined as $(1\pm\gamma_5)/2$, respectively. In (1), C_X are the coefficients of the four-Fermi interactions with $X = \mathrm{LL, LR, RL, RR}$ describing vector-, $X = \mathrm{LRLR, RLLR, LRRL, RLRL}$ scalar- and $X = \mathrm{T, TE}$ tensor-type interactions. We note that several of the Wilson coefficients in (1) do already exist in the SM: in the SM, $C_{\rm LL}$ and $C_{\rm LR}$ are in the form $C_9^{\rm eff} - C_{10}$ and $C_9^{\rm eff} + C_{10}$ for the $b \to s\ell^+\ell^-$ decay, while the coefficients $C_{\rm SL}$ and $C_{\rm BR}$ correspond to $-2m_s C_7^{\rm eff}$ and $-2m_b C_7^{\rm eff}$, respectively. Therefore, writing

$$\begin{aligned}
C_{\rm LL}^{\rm tot} &= C_9^{\rm eff} - C_{10} + C_{\rm LL}\,,\\
C_{\rm LR}^{\rm tot} &= C_9^{\rm eff} + C_{10} + C_{\rm LR}\,,
\end{aligned}$$

we see that $C_{\rm LL}^{\rm tot}$ and $C_{\rm LR}^{\rm tot}$ contain contributions from the SM and also from new physics.

Having established the general form of the effective Hamiltonian, we proceed to calculate the matrix element of the $B_s \to \gamma\,\ell^+\ell^-$ decay. This exclusive decay can receive short-distance contributions from the box, Z, and photon penguin diagrams for the $b \to s$ transition by attaching an additional photon line to any internal or external lines. As pointed out before [7,8], contributions coming from the release of the free photon from any charged internal line will be suppressed by a factor of m_b^2/M_W^2 and we neglect them in the following analysis. When a photon is released from the initial quark lines it contributes to the so-called "structure dependent" (SD) part of the amplitude, $\mathcal{M}_{\rm SD}$. Then it follows from (1) that, in order to calculate $\mathcal{M}_{\rm SD}$, the matrix elements needed and their definitions in terms of the various form factors are as follows [7,20]:

$$\begin{aligned}
&\langle\gamma(k)\,|\bar{s}\gamma_\mu(1\mp\gamma_5)b|\,B(p_B)\rangle \\
&= \frac{e}{m_B^2}\{\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}q^\lambda k^\sigma g(q^2) \\
&\qquad \pm\mathrm{i}\,[\varepsilon^{*\mu}(kq) - (\varepsilon^* q)k^\mu]\,f(q^2)\}\,, \qquad (2)
\end{aligned}$$

$$\begin{aligned}
&\langle\gamma(k)\,|\bar{s}\sigma_{\mu\nu}b|\,B(p_B)\rangle \\
&= \frac{e}{m_B^2}\epsilon_{\mu\nu\lambda\sigma}\left[G\varepsilon^{*\lambda}k^\sigma + H\varepsilon^{*\lambda}q^\sigma + N(\varepsilon^* q)q^\lambda k^\sigma\right]\,, \qquad (3)\\
&\langle\gamma(k)\,|\bar{s}(1\mp\gamma_5)b|\,B(p_B)\rangle = 0\,, \qquad (4)\\
&\langle\gamma\,|\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu b|\,B(p_B)\rangle = \frac{e}{m_B^2}\mathrm{i}\,\epsilon_{\mu\nu\alpha\beta}q^\nu\varepsilon^{\alpha *}k^\beta G\,, \qquad (5)
\end{aligned}$$

and

$$\begin{aligned}
&\langle\gamma(k)\,|\bar{s}\mathrm{i}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\,B(p_B)\rangle \\
&= \frac{e}{m_B^2}\Big\{\epsilon_{\mu\alpha\beta\sigma}\,\varepsilon^{\alpha *}q^\beta k^\sigma g_1(q^2) \\
&\qquad +\mathrm{i}\,\left[\varepsilon_\mu^*(qk) - (\varepsilon^* q)k_\mu\right]f_1(q^2)\Big\}\,, \qquad (6)
\end{aligned}$$

where ε_μ^* and k_μ are the four vector polarization and four momentum of the photon, respectively, q is the momentum transfer, p_B is the momentum of the B-meson, and G, H and N can be expressed in terms of the form factors g_1 and f_1 by using (3), (5) and (6). The matrix element describing the structure dependent part can be obtained from (2)–(6):

$$\begin{aligned}
\mathcal{M}_{\rm SD} &= \frac{\alpha G_{\rm F}}{4\sqrt{2}\,\pi}V_{tb}V_{ts}^*\frac{e}{m_B^2} \\
&\times \left\{\bar{\ell}\gamma^\mu(1-\gamma_5)\ell\right. \\
&\times \left[A_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha k^\beta + \mathrm{i}\,A_2(\varepsilon_\mu^*(kq) - (\varepsilon^* q)k_\mu)\right] \\
&+ \bar{\ell}\gamma^\mu(1+\gamma_5)\ell \qquad (7)\\
&\times \left[B_1\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha k^\beta + \mathrm{i}\,B_2(\varepsilon_\mu^*(kq) - (\varepsilon^* q)k_\mu)\right] \\
&+ \mathrm{i}\,\epsilon_{\mu\nu\alpha\beta}\bar{\ell}\sigma^{\mu\nu}\ell\,\left[G\varepsilon^{*\alpha}k^\beta + H\varepsilon^{*\alpha}q^\beta + N(\varepsilon^* q)q^\alpha k^\beta\right] \\
&+ \mathrm{i}\,\bar{\ell}\sigma_{\mu\nu}\ell\big[G_1(\varepsilon^{*\mu}k^\nu - \varepsilon^{*\nu}k^\mu) + H_1(\varepsilon^{*\mu}q^\nu - \varepsilon^{*\nu}q^\mu) \\
&+ N_1(\varepsilon^* q)(q^\mu k^\nu - q^\nu k^\mu)\big]\big\}\,,
\end{aligned}$$

where

$$A_1 = \frac{1}{q^2}(C_{\rm BR} + C_{\rm SL})g_1 + (C_{\rm LL}^{\rm tot} + C_{\rm RL})g\,,$$

$$
\begin{aligned}
A_2 &= \frac{1}{q^2}(C_{\rm BR} - C_{\rm SL}) f_1 + (C_{\rm LL}^{\rm tot} - C_{\rm RL}) f \ , \\
B_1 &= \frac{1}{q^2}(C_{\rm BR} + C_{\rm SL}) g_1 + (C_{\rm LR}^{\rm tot} + C_{\rm RR}) g \ , \\
B_2 &= \frac{1}{q^2}(C_{\rm BR} - C_{\rm SL}) f_1 + (C_{\rm LR}^{\rm tot} - C_{\rm RR}) f \ , \\
G &= 4C_{\rm T} g_1 \quad , \quad N = -4C_{\rm T}\frac{1}{q^2}(f_1+g_1) \ , \\
H &= N(qk) \quad , \quad G_1 = -8C_{\rm TE} g_1 \ , \\
N_1 &= 8C_{\rm TE}\frac{1}{q^2}(f_1+g_1) \quad , \quad H_1 = N_1(qk) \ .
\end{aligned}
$$

When a photon is radiated from the lepton line we get the so-called "internal Bremsstrahlung" (IB) contribution, $\mathcal{M}_{\rm IB}$. Using the expressions

$$
\begin{aligned}
\langle 0|\bar{s}\gamma_\mu\gamma_5 b|B(p_B)\rangle &= -\,{\rm i} f_B p_{B\mu} \ , \\
\langle 0|\bar{s}\sigma_{\mu\nu}(1+\gamma_5) b|B(p_B)\rangle &= 0 \ ,
\end{aligned}
$$

and conservation of the vector current, we get

$$
\begin{aligned}
\mathcal{M}_{\rm IB} &= \frac{\alpha G_{\rm F}}{4\sqrt{2}\,\pi} V_{tb}V_{ts}^* e f_B {\rm i} \left\{ F\,\bar{\ell}\left(\frac{\not{\varepsilon}^*\not{p}_B}{2p_1k} - \frac{\not{p}_B\not{\varepsilon}^*}{2p_2k}\right)\gamma_5 \ell \right. \quad (8) \\
&+ \left. F_1\,\bar{\ell}\left[\frac{\not{\varepsilon}^*\not{p}_B}{2p_1k} - \frac{\not{p}_B\not{\varepsilon}^*}{2p_2k} + 2m_\ell\left(\frac{1}{2p_1k}+\frac{1}{2p_2k}\right)\not{\varepsilon}^*\right]\ell \right\} \ ,
\end{aligned}
$$

where

$$
\begin{aligned}
F &= 2m_\ell(C_{\rm LR}^{\rm tot} - C_{\rm LL}^{\rm tot} + C_{\rm RL} - C_{\rm RR}) \\
&+ \frac{m_B^2}{m_b}(C_{\rm LRLR} - C_{\rm RLLR} - C_{\rm LRRL} + C_{\rm RLRL}) \ , \\
F_1 &= \frac{m_B^2}{m_b}(C_{\rm LRLR} - C_{\rm RLLR} + C_{\rm LRRL} - C_{\rm RLRL}) \ . \quad (9)
\end{aligned}
$$

Finally, the total matrix element for the $B_s \to \gamma\,\ell^+\ell^-$ decay is obtained as a sum of the $\mathcal{M}_{\rm SD}$ and $\mathcal{M}_{\rm IB}$ terms,

$$
\mathcal{M} = \mathcal{M}_{\rm SD} + \mathcal{M}_{\rm IB}. \quad (10)
$$

The next task is the calculation of the differential decay rate of $B_s \to \gamma\,\ell^+\ell^-$ decay as a function of the dimensionless parameter $x = 2E_\gamma/m_B$, where E_γ is the photon energy. In the center of mass (c.m.) frame of the dileptons $\ell^+\ell^-$, where we take $z = \cos\theta$, and θ is the angle between the momentum of the B_s-meson and that of ℓ^-, the double differential decay width is found to be

$$
\frac{{\rm d}\Gamma}{{\rm d}x\,{\rm d}z} = \frac{1}{(2\pi)^3 64}\, x\, v\, m_B\, |\mathcal{M}|^2 \ , \quad (11)
$$

with

$$
|\mathcal{M}|^2 = |\mathcal{M}_{\rm SD}|^2 + |\mathcal{M}_{\rm IB}|^2 + 2{\rm Re}(\mathcal{M}_{\rm SD}\mathcal{M}_{\rm IB}^*), \quad (12)
$$

where $v = \sqrt{1-\frac{4r}{1-x}}$ and $r = m_\ell^2/m_B^2$. We note that the $|\mathcal{M}_{\rm IB}|^2$ term has an infrared singularity due to the emission of soft photons. In order to obtain a finite result, we follow the approach described in [8] and impose a cut on the photon energy, i.e., we require $E_\gamma \geq 25\,{\rm MeV}$, which corresponds to the detectection of only hard photons experimentally. This cut requires that $E_\gamma \geq \delta\, m_B/2$ with $\delta = 0.01$.

In such a radiative decay, the final state photon can emerge with a definite polarization and there follows the question of how sensitive the branching ratio is to the new Wilson coefficients when the photon is in the positive or negative helicity state. To find an answer to this question, we evaluate $\frac{{\rm d}\Gamma(\varepsilon^*=\varepsilon_1)}{{\rm d}x}$ and $\frac{{\rm d}\Gamma(\varepsilon^*=\varepsilon_2)}{{\rm d}x}$ for $B_s \to \gamma\,\ell^+\ell^-$ decay, in the c.m. frame of $\ell^+\ell^-$, in which the four-momenta and polarization vectors, ε_1 and ε_2, are as follows:

$$
\begin{aligned}
P_B &= (E_B, 0, 0, E_k) \ , \quad k = (E_k, 0, 0, E_k) \ , \\
p_1 &= (p, 0, p\sqrt{1-z^2}, -pz) \, , \\
p_2 &= (p, 0, -p\sqrt{1-z^2}, pz) \ , \\
\varepsilon_1 &= (0, 1, {\rm i}, 0)/\sqrt{2} \ , \quad \varepsilon_2 = (0, 1, -{\rm i}, 0)/\sqrt{2} \ , \quad (13)
\end{aligned}
$$

where $E_B = m_B(2-x)/2\sqrt{1-x}$, $E_k = m_B x/2\sqrt{1-x}$, and $p = m_B\sqrt{1-x}/2$. Using the above forms, we obtain

$$
\frac{{\rm d}\Gamma(\varepsilon^*=\varepsilon_i)}{{\rm d}x} = \left|\frac{\alpha G_{\rm F}}{4\sqrt{2}\,\pi} V_{tb}V_{ts}^*\right|^2 \frac{\alpha}{(2\,\pi)^3}\frac{\pi}{4}\, m_B\, \Delta(\varepsilon_i), \quad (14)
$$

where

$$
\begin{aligned}
\Delta(\varepsilon_i) &= \frac{vx}{3}\{4\,x((8r+x)\,|H_1|^2 - (4r-x)\,|H|^2) \\
&- 6m_\ell(1-x)^2\,{\rm Im}\,[(A_2 \pm A_1 + B_2 \pm B_1)G_1^*] \\
&+ \frac{2}{x}(1-x)^2(2r+x)(|G_1|^2 + |G|^2 \pm 2\,{\rm Im}\,[-G_1G^*])) \\
&- 12m_\ell(1-x)x\,{\rm Im}\,[(A_2 \pm A_1 + B_2 \pm B_1)H_1^*] \\
&\pm 4(1-x)((8r+x)\,{\rm Im}\,[GH_1^*] + (4r-x)\,{\rm Im}\,[G_1H^*]) \\
&+ 6m_\ell^2(1-x)^2\,{\rm Re}\,[(A_1 \pm A_2)(B_1 \pm B_2)] \\
&+ m_B^2(1-x)^2(x-r) \\
&\times (|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2 \pm 2\,{\rm Re}\,[A_1A_2^* + B_1B_2^*]) \\
&- 6m_\ell(1-x)^2\,{\rm Re}\,[(A_2 \pm A_1 + B_2 \pm B_1)G^*] \\
&+ 4(1-x)((8r+x)\,{\rm Re}\,[G_1H_1^*] - (4r-x)\,{\rm Re}[GH^*])\} \\
&+ \frac{2x}{(1-x)^2} f_B^2\{(-2vx + (1-4r+x^2)\ln[u])\,|F|^2 \\
&\pm 2(1-x)(2vx - (1-4r+x)\ln[u])\,{\rm Re}\,[FF_1^*] \\
&+ (2vx(4r-1) \\
&+ (1+16r^2+x^2-4r(1+2x))\ln[u]\,|F_1|^2)\} \\
&+ 2xf_B\,\{\pm(vx + 2r\ln[u])\,{\rm Im}\,[-FH_1^*] \\
&\pm m_\ell(1-x)\ln[u]\,{\rm Re}\,[(A_2 \pm A_1 + B_2 \pm B_1)F^*]
\end{aligned}
$$

$$
\begin{aligned}
&- m_\ell(2vx + (1-4r-x)\ln[u]) \\
&\times \mathrm{Re}\,[(A_2 \pm A_1 + B_2 \pm B_1)F_1^*] \\
&- 2(v - 2r\ln[u])\,\mathrm{Im}\,[(-F_1 \pm F)(G_1^* \pm G^*)] \\
&\pm 2(vx - 2r\ln[u])\,\mathrm{Re}\,[F_1 H^*] \\
&+ \frac{2}{(1-x)} \\
&\times ((vx(1+x) + 2r(1-3x)\ln[u])\,\mathrm{Im}\,[F_1 H_1^*] \\
&- (1+x)(vx - 2r\ln[u])\,\mathrm{Re}\,[F_1 H^*])\}\,, \qquad (15)
\end{aligned}
$$

where $+(-)$ is for $i = 1(2)$ and $u = 1 + v/1 - v$.

The effects of the polarized photon can be also studied through the variable "photon polarization asymmetry" [11]:

$$
\begin{aligned}
H(x) &= \frac{\frac{\mathrm{d}\Gamma(\varepsilon^*=\varepsilon_1)}{\mathrm{d}x} - \frac{\mathrm{d}\Gamma(\varepsilon^*=\varepsilon_2)}{\mathrm{d}x}}{\frac{\mathrm{d}\Gamma(\varepsilon^*=\varepsilon_1)}{\mathrm{d}x} + \frac{\mathrm{d}\Gamma(\varepsilon^*=\varepsilon_2)}{\mathrm{d}x}} \\
&= \frac{\Delta(\varepsilon_1) - \Delta(\varepsilon_2)}{\Delta_0}\,, \qquad (16)
\end{aligned}
$$

where

$$
\begin{aligned}
&\Delta(\varepsilon_1) - \Delta(\varepsilon_2) \\
&= \tfrac{4}{3}x^2 v\Bigg\{\frac{2x(1+2r-x)}{(-1+x)}\,\mathrm{Im}\,[G_1 G^*] \\
&- 3m_\ell x(\mathrm{Im}\,[(A_1+B_1)G_1^*] + \mathrm{Re}\,[(A_2+B_2)G^*])\Bigg\} \\
&- 6m_\ell(1-x)((\mathrm{Im}\,[(A_1+B_1)H_1^*]) \\
&+ 2((1+8r-x)\,\mathrm{Im}\,[GH_1^*] - (1-4r-x)\,\mathrm{Im}\,[G_1 H^*]) \\
&+ m_B^2 x(3r(\mathrm{Re}\,[A_2 B_1^* + A_1 B_2^*] \\
&+ (1-r-x)\,\mathrm{Re}\,[B_1 B_2^* + A_1 A_2^*]) \\
&+ 8f_B^2(2v(1-x) - (2-4r-x)\ln[u]) \\
&+ 4f_B x\{2(v(x-1) - 2r\ln[u])\,\mathrm{Im}\,[FH_1^*] \\
&+ m_\ell x\ln[u]\,\mathrm{Re}\,[(A_2+B_2)F^*] + m_\ell(2v(x-1) \\
&+ (4r-x)\ln[u])\,\mathrm{Re}\,[(A_1+B_1)F_1^*] \\
&+ 2(v - 2r\ln[u])\,\mathrm{Re}\,[F_1 G^*] - \mathrm{Im}\,[FG_1^*] \\
&+ 2(v(1-x) - 2r\ln[u])\,\mathrm{Re}\,[F_1 H^*]\} \qquad (17)
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_0 = &\Bigg\{x^3 v\Big(4m_\ell\ \mathrm{Re}\,([A_1+B_1]G^*) \\
&- 4m_B^2 r\ \mathrm{Re}\,(A_1 B_1^* + A_2 B_2^*) \\
&- 4\left[|H_1|^2(1-x) + \mathrm{Re}\,(G_1 H_1^*)\,x\right]\frac{(1+8r-x)}{x^2} \\
&- 4\left[|H|^2(1-x) + \mathrm{Re}\,(GH^*)\,x\right]\frac{(1-4r-x)}{x^2} \\
&+ \tfrac{1}{3}m_B^2\left[2\ \mathrm{Re}\,(GN^*) + m_B^2\,|N|^2(1-x)\right](1-4r-x) \\
&+ \tfrac{1}{3}m_B^2\left[2\ \mathrm{Re}\,(G_1 N_1^*) + m_B^2\,|N_1|^2(1-x)\right](1+8r-x) \\
&- \tfrac{2}{3}m_B^2\left(|A_1|^2 + |A_2|^2 + |B_1|^2 + |B_2|^2\right)(1-r-x) \\
&- \tfrac{4}{3}\left(|G|^2 + |G_1|^2\right)\frac{(1+2r-x)}{(1-x)} \\
&+ 2m_\ell\ \mathrm{Im}([A_2+B_2] \\
&\times\left[6H_1^*(1-x) + 2G_1^* x - m_B^2\,N_1^* x(1-x)\right])\frac{1}{x}\Big) \\
&+ 4f_B\,(2v\Big[\mathrm{Re}\,(FG^*)\,\frac{1}{(1-x)} - \mathrm{Re}\,(FH^*) \\
&+ m_B^2\ \mathrm{Re}\,(FN^*) + m_\ell\ \mathrm{Re}\,([A_2+B_2]F_1^*)\Big]x(1-x) \\
&+ \ln[u]\Big[m_\ell\ \mathrm{Re}\,([A_2+B_2]F_1^*)\ x(x-4r) \\
&+ 2\ \mathrm{Re}\,(FH^*)\,[1-x+2r(x-2)] \\
&- 4rx\ \mathrm{Re}\,(FG^*) + m_B^2\ \mathrm{Re}\,(FN^*)\ x(x-1) \\
&- m_\ell\ \mathrm{Re}\,([A_1+B_1]F^*)\ x^2\Big] \\
&+ 2\left[m_B^2\,\mathrm{Im}\,(F_1 N_1^*)\,(v(1-x) + (x-1-2rx)\ln[u])\right. \\
&+ \mathrm{Im}\,(F_1 H_1^*)\left(v(x-1) + \frac{1-x-4r(2x-1)\ln[u]}{x}\right) \\
&+ \left.\mathrm{Im}\,(F_1 G_1^*)\,(v - 2r\ln[u])\right] \\
&+ 4f_B^2\Bigg(2v\left(|F|^2 + (1-4r)\,|F_1|^2\right)\frac{(1-x)}{x} \\
&+ \ln[u]\Bigg[|F|^2\left(2 + \frac{4r}{x} - \frac{2}{x} - x\right) \qquad (18) \\
&+ |F_1|^2\left(2(1-4r) - \frac{2\left(1-6r+8r^2\right)}{x} - x\right)\Bigg]\Bigg)\Bigg\}.
\end{aligned}
$$

The expression in (17) agrees with [11] for the SM case with neutral Higgs contributions.

3 Numerical analysis and discussion

We present here our numerical analysis of the branching ratios (BR) and the photon polarization asymmetries (H) for the $B_s \to \gamma\ell^+\ell^-$ decays with $\ell = \mu, \tau$. We first give the input parameters used in our numerical analysis:

$$
\begin{aligned}
&m_B = 5.28\,\mathrm{GeV}\,,\ m_b = 4.8\,\mathrm{GeV}\,,\ m_\mu = 0.105\,\mathrm{GeV}\,, \\
&m_\tau = 1.78\,\mathrm{GeV}\,,\ f_B = 0.2\,\mathrm{GeV}\,,\ |V_{tb}V_{ts}^*| = 0.045\,, \\
&\alpha^{-1} = 137\,,\ G_\mathrm{F} = 1.17\times 10^{-5}\,\mathrm{GeV}^{-2}, \qquad (19)
\end{aligned}
$$

Table 1. Values of the SM Wilson coefficients at the $\mu \sim m_b$ scale

C_1	C_2	C_3	C_4	C_5	C_6	$C_7^{\rm eff}$	C_9	C_{10}
−0.248	+1.107	+0.011	−0.026	+0.007	−0.031	−0.313	+4.344	−4.624

$$\tau_{B_s} = 1.54 \times 10^{-12}\,\mathrm{s}\ ,\ C_9^{\rm eff} = 4.344\ ,\ C_{10} = -4.669.$$

As for the values of the new Wilson coefficients, which are responsible for the new physics beyond the SM, they are free parameters in this work. However, it is possible to establish ranges out of the experimentally measured branching ratios of the semileptonic rare B-meson decays $B \to K\,\ell^+\ell^-$ and $B \to K^*\,\ell^+\ell^-$, recently announced by the BaBar Collaboration [21]:

$$\mathrm{BR}(B \to K\,\ell^+\ell^-) = (0.78^{+0.24+0.11}_{-0.20-0.18}) \times 10^{-6}\ ,$$

$$\mathrm{BR}(B \to K^*\,\ell^+\ell^-) = (1.68^{+0.68}_{-0.58} \pm 0.28) \times 10^{-6}\ .$$

The $B \to K\,\ell^+\ell^-$ decay has been also observed by the BELLE Collaboration [22] with the branching ratio BR $(B \to K\,\ell^+\ell^-) = (0.75^{+0.25}_{-0.21} \pm 0.09) \times 10^{-6}$. In addition, there is now available an upper bound of pure leptonic rare B decays in the $B^0 \to \mu^+\mu^-$ mode [23]:

$$\mathrm{BR}(B^0 \to \mu^+\mu^-) \le 2.0 \times 10^{-7}\ .$$

Using these available experimental data we find that the right order of magnitude for the new Wilson coefficients is in the range $-4 \le C_X \le 4$, assuming that they are real. We further note that some of the new Wilson coefficients in (1) appear in some well known models beyond the SM, like some MSSM scenarios, and in the literature there exist studies to establish the ranges out of constraints under various precision measurements for these coefficients (see, e.g., [24]). Our choice for the range of the new Wilson coefficients above are also in agreement with these calculations.

It should be noted here that the value of the Wilson coefficient $C_9^{\rm eff}$ above corresponds only to the short-distance (SD) contributions. $C_9^{\rm eff}$ also receives long-distance (LD) contributions due to the conversion of the real $\bar{c}c$ into the lepton pair $\ell^+\ell^-$ and they are usually absorbed into a redefinition of the short-distance Wilson coefficients:

$$C_9^{\rm eff}(\mu) = C_9(\mu) + Y(\mu)\ , \tag{20}$$

where

$$\begin{aligned} &Y(\mu) \\ &= Y_{\rm reson} + h(z,s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) \\ &+ 3C_5(\mu) + C_6(\mu)] - \tfrac{1}{2}h(1,s) \\ &\times (4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ &- \tfrac{1}{2}h(0,s)\,[C_3(\mu) + 3C_4(\mu)] \\ &+ \tfrac{2}{9}\,(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu))\ ; \end{aligned} \tag{21}$$

$z = m_c/m_b$, $s = q^2/mB^2$ and the values of the individual Wilson coefficients are listed in Table 1. The functions $h(z,s)$ arise from the one loop contributions of the four quark operators O_1,...,O_6 and their explicit forms can be found in [25]. It is possible to parameterize the resonance $\bar{c}c$ contribution $Y_{\rm reson}(s)$ in (21) using a Breit–Wigner shape with normalizations fixed by data which is given by [26]

$$\begin{aligned} Y_{\rm reson}(s) = -\frac{3}{\alpha_{em}^2}\kappa \sum_{V_i = J/\psi,\psi',\ldots} \frac{\pi\Gamma(V_i \to \ell^+\ell^-)m_{V_i}}{sm_B^2 - m_{V_i} + im_{V_i}\Gamma_{V_i}} \\ \times\big[(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) \\ +3C_5(\mu) + C_6(\mu))\big]\ , \end{aligned} \tag{22}$$

where the phenomenological parameter κ is usually taken as ~ 2.3.

To make some numerical predictions, we also need the explicit forms of the form factors g, f, g_1 and f_1. They are calculated in the framework of light-cone QCD sum rules in [7, 20], and also in [27] in terms of the two parameters $F(0)$ and m_F. In our work we have used the results of [7], in which the q^2 dependences of the form factors are given by

$$g(q^2) = \frac{1\,\mathrm{GeV}}{\left(1 - \frac{q^2}{5.6^2}\right)^2}\ ,\ f(q^2) = \frac{0.8\,\mathrm{GeV}}{\left(1 - \frac{q^2}{6.5^2}\right)^2}\ ,$$

$$g_1(q^2) = \frac{3.74\,\mathrm{GeV}^2}{\left(1 - \frac{q^2}{40.5}\right)^2}\ ,\ f_1(q^2) = \frac{0.68\,\mathrm{GeV}^2}{\left(1 - \frac{q^2}{30}\right)^2}\ .$$

We present the results of our analysis in a series of figures. Before their discussion we give our SM predictions for the unpolarized BRs without LD effects, for reference:

$$\mathrm{BR}(B_s \to \gamma\mu^+\mu^-) = 1.52 \times 10^{-8}\ ,$$

$$\mathrm{BR}(B_s \to \gamma\tau^+\tau^-) = 1.19 \times 10^{-8}\ ,$$

which are in good agreement with the results of [15].

In Figs. 1 and 2, we present the dependence of $\mathrm{BR}^{(1)}$ and $\mathrm{BR}^{(2)}$ for $B_s \to \gamma\mu^+\mu^-$ decay on the new Wilson coefficients, where the superscripts (1) and (2) correspond to the positive and negative helicity states of photon, respectively. From these figures we see that $\mathrm{BR}^{(1)}$ and $\mathrm{BR}^{(2)}$ are more sensitive to all types of scalar interactions as compared to the vector and tensor types, receiving the maximum contribution from the one with coefficient $C_{\rm RLRL}$ and $C_{\rm LRLR}$, respectively. From Fig. 2, we also observe that the dependence of $\mathrm{BR}^{(2)}$ on all the new Wilson coefficients is symmetric with respect to the zero point, while for $\mathrm{BR}^{(1)}$, this symmetry is slightly lifted for the vector-type interactions (Fig. 1). It follows that $\mathrm{BR}^{(2)}$ decreases

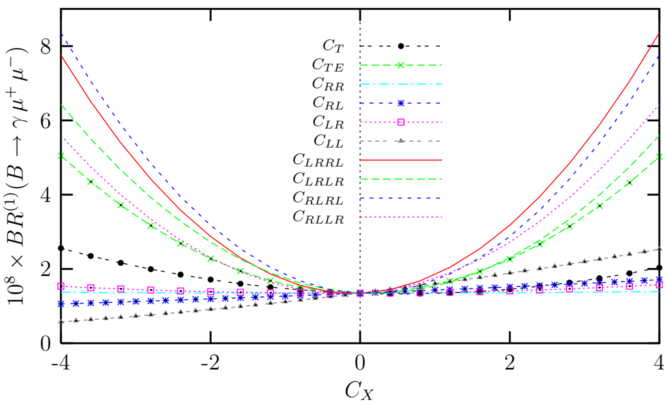

Fig. 1. The dependence of the integrated branching ratio for the $B_s \to \gamma \mu^+ \mu^-$ decay with the photon in the positive helicity state on the new Wilson coefficients with LD effects

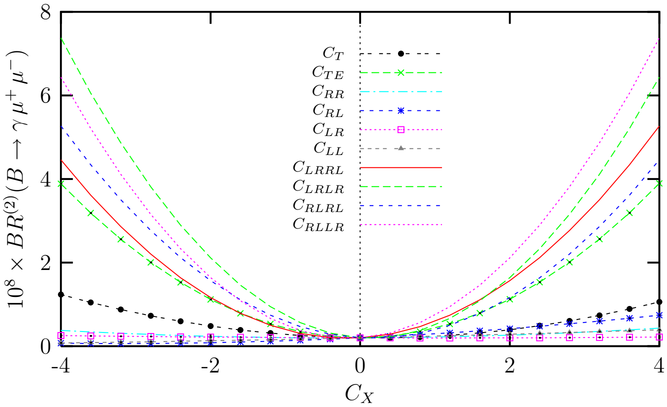

Fig. 2. The same as Fig. 1, but with the photon in the negative helicity state

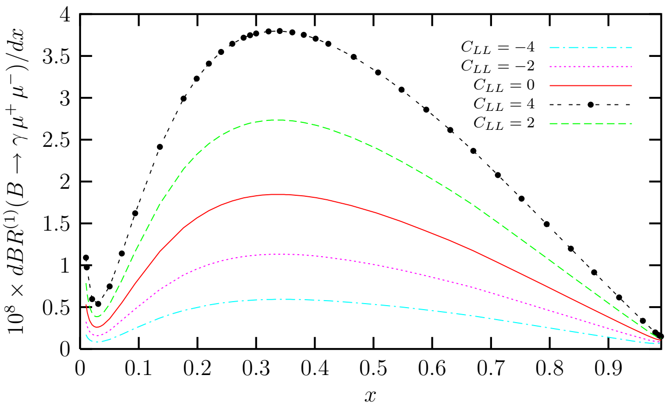

Fig. 3. The dependence of the differential branching ratio for the $B_s \to \gamma \mu^+ \mu^-$ decay with the photon in the positive helicity state on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of vector interaction with coefficient $C_{\rm LL}$ without LD effects

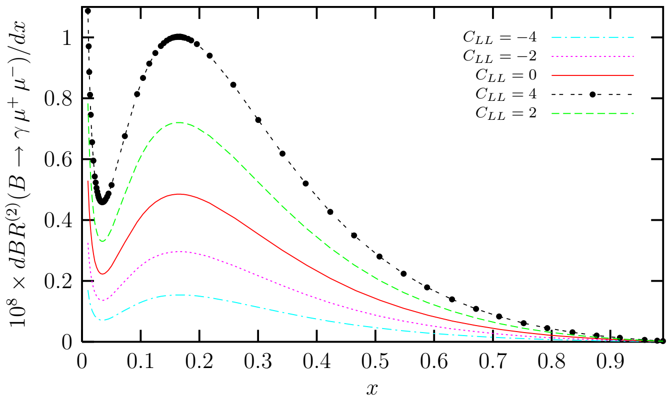

Fig. 4. The same as Fig. 3, but with the photon in the negative helicity state

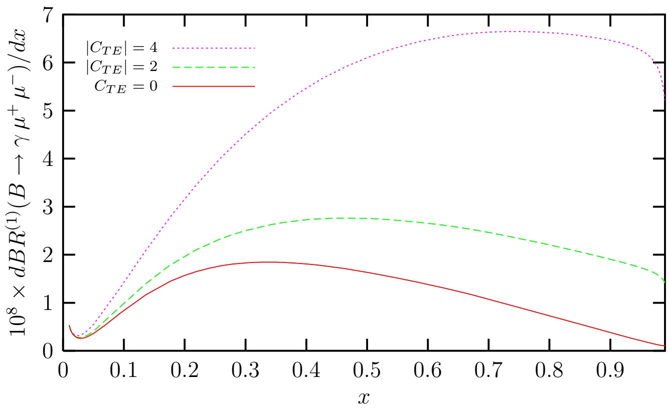

Fig. 5. The dependence of the differential branching ratio for the $B_s \to \gamma \mu^+ \mu^-$ decay with the photon in the positive helicity state on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of the tensor interaction with coefficient $C_{\rm TE}$ without LD effects

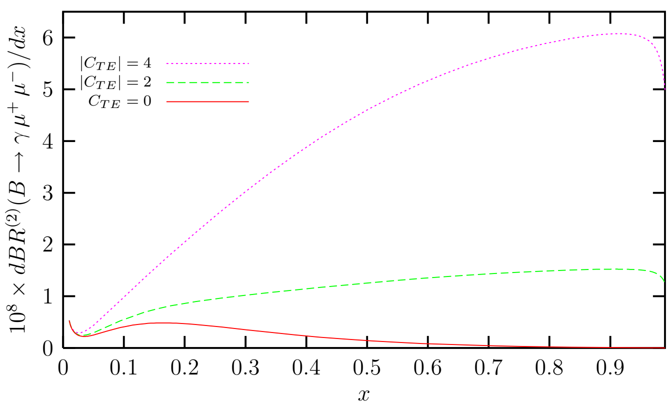

Fig. 6. The same as Fig. 5, but with the photon in the negative helicity state

in the region $-4 \le C_X \le 0$ and tends to increase in between $0 \le C_X \le +4$. $\mathrm{BR}^{(1)}$ exhibits a similar behavior, except for the vector interactions with coefficients $C_{\rm LL}$, $C_{\rm RL}$ and $C_{\rm LR}$: it is almost insensitive to the existence of vector-type $C_{\rm LR}$ interactions and slightly increases with increasing values of $C_{\rm LL}$ and $C_{\rm RL}$, receiving a value lower than the SM one between -4 and 0.

The differential branching ratio can also give useful information on new physics effects. Therefore, in Figs. 3–8 we present the dependence of the differential branching ratio with a polarized photon for the $B_s \to \gamma \mu^+ \mu^-$ decay

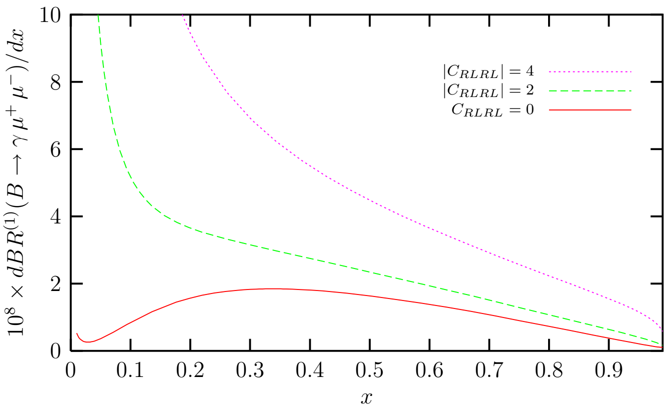

Fig. 7. The dependence of the differential branching ratio for the $B_s \to \gamma\, \mu^+ \mu^-$ decay with the photon in the positive helicity state on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of the scalar interaction with coefficient $C_{\rm RLRL}$ without LD effects

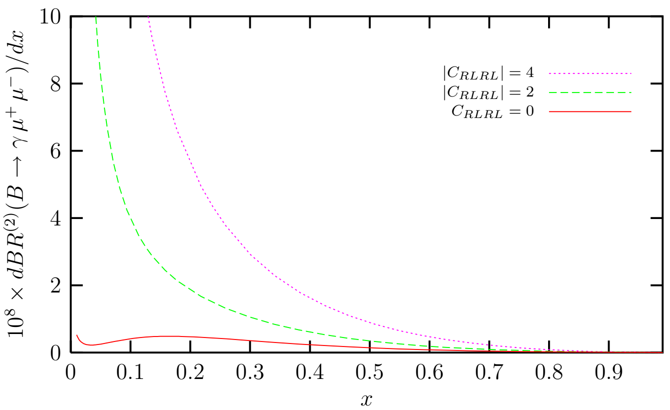

Fig. 8. The same as Fig. 7, but with the photon in the negative helicity state

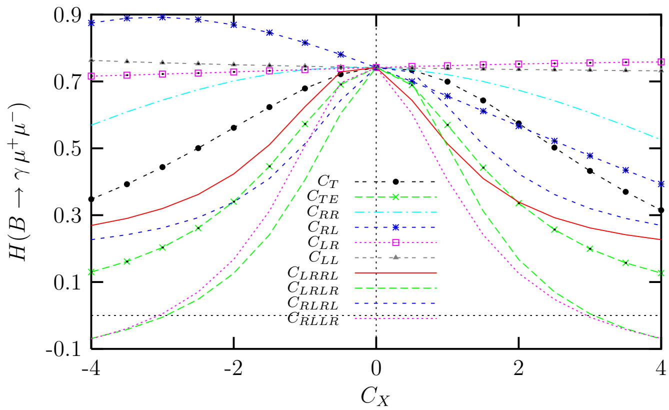

Fig. 9. The dependence of the integrated photon polarization asymmetry for the $B_s \to \gamma\, \mu^+ \mu^-$ decay on the new Wilson coefficients with LD effects

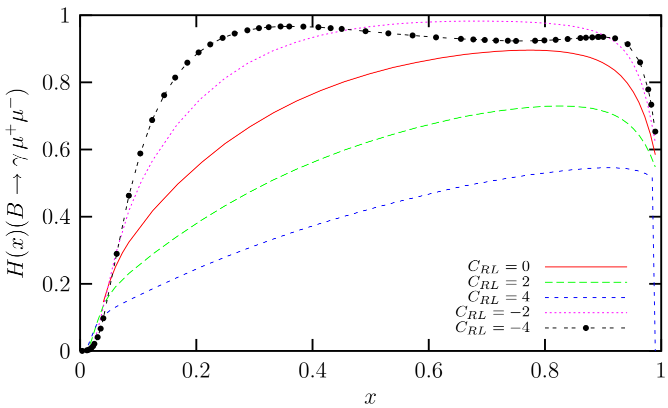

Fig. 10. The dependence of the differential photon polarization asymmetry for the $B_s \to \gamma\, \mu^+ \mu^-$ decay on the dimensionless variable $x = 2E_\gamma/m_B$ for different values of $C_{\rm RL}$ without LD effects

on the dimensionless variable $x = 2E_\gamma/m_B$ at different values of vector, tensor and scalar interactions with coefficients $C_{\rm LL}$, $C_{\rm TE}$ and $C_{\rm RLRL}$. We observe that tensor- (scalar-) type interactions change the spectrum near the large- (small-) recoil limit, $x \to 1$ ($x \to 0$), as seen from Figs. 5 and 6 (Figs. 7 and 8). However, the vector-type interactions increase the spectrum in the center of the phase space and do not change it at the large- or small-recoil limit (Figs. 3 and 4). We also see from Figs. 3 and 4 that when $C_{\rm LL} > 0$, the related vector interaction gives a constructive contribution to the SM result, but for negative values of $C_{\rm LL}$ the contribution is destructive. Therefore, it is possible to get information on the sign of new Wilson coefficients from a measurement of the differential branching ratio.

From Figs. 1–8, we also see that the branching ratios with a positive helicity photon are greater than those with a negative helicity one. To see this we rewrite (15) for the SM in the limit $m_\ell \to 0$,

$$\Delta(\varepsilon_i) = \frac{m_B^2}{3} x^2 (-1+x)^2 \tag{23}$$

$$\times \left\{ \left| (C_9^{\rm eff} - C_{10})(g \pm f) - \frac{2C_7}{(1-x)m_B^2} m_b (g_1 \pm f_1) \right|^2 + \left| (C_9^{\rm eff} + C_{10})(g \pm f) - \frac{2C_7}{(1-x)m_B^2} m_b (g_1 \pm f_1) \right|^2 \right\},$$

where $+(-)$ is for $i = 1(2)$. It obviously follows that ${\rm BR}^{(1)} > {\rm BR}^{(2)}$. We note that this fact can be seen more clearly from the comparison of the differential BRs for the (1) and (2) cases for the vector interactions with the coefficient $C_{\rm LL}$, given in Figs. 3 and 4, where ${\rm dBR}^{(1)}/{\rm d}x$ is larger by about four times as compared to ${\rm dBR}^{(2)}/{\rm d}x$.

In addition to the total and differential branching ratios, for radiative decays like ours, studying the effects of a polarized photon may provide useful information on the new Wilson coefficients. For this purpose, we present the dependence of the integrated photon polarization asymmetry H for $B_s \to \gamma\, \mu^+ \mu^-$ decay on the new Wilson coefficients in Figs. 9 and 10. We see from Fig. 9 that the spectrum of H is almost symmetrical with respect to the zero point for all the new Wilson coefficients, except the $C_{\rm RL}$. The coefficient $C_{\rm RL}$, when it is between -2 and 0,

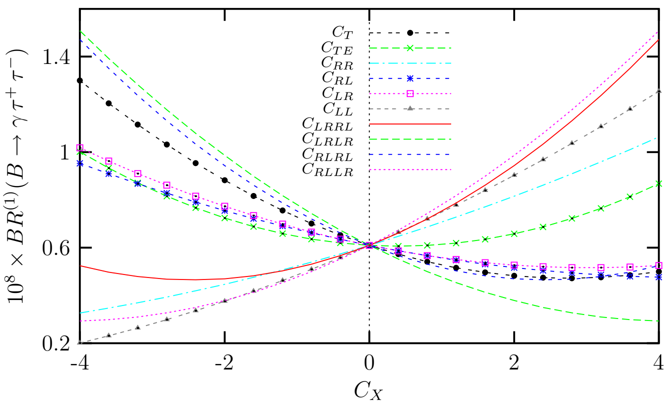

Fig. 11. The dependence of the integrated branching ratio for the $B_s \to \gamma\,\tau^+\tau^-$ decay with the photon in the positive helicity state on the new Wilson coefficients with LD effects

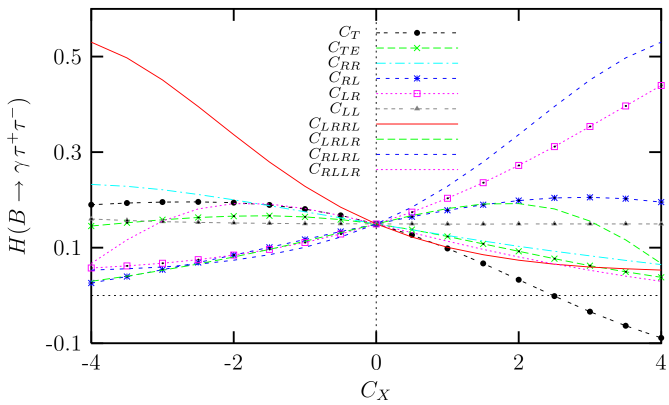

Fig. 13. The dependence of the integrated photon polarization asymmetry for the $B_s \to \gamma\,\tau^+\tau^-$ decay on the new Wilson coefficients with LD effects

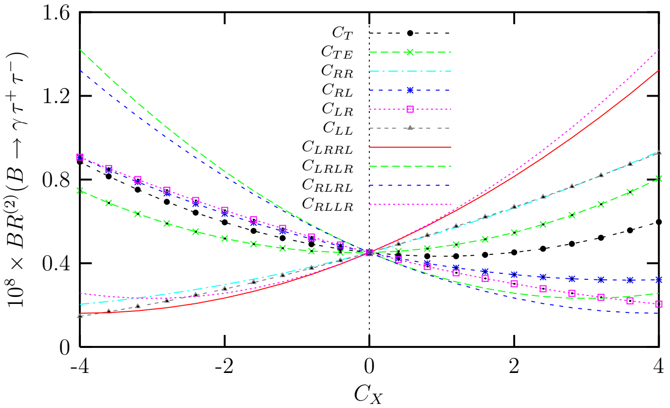

Fig. 12. The same as Fig. 11, but with the photon in the negative helicity state

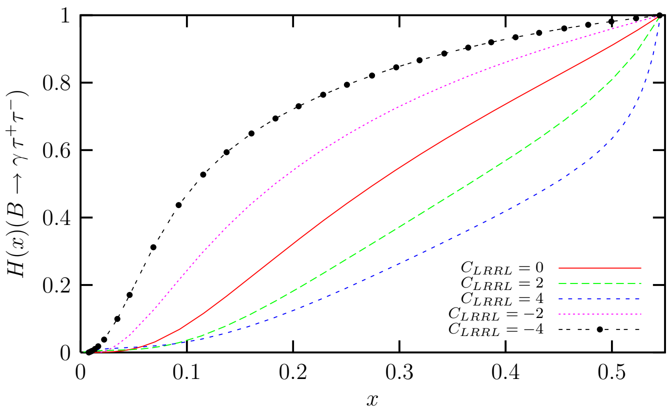

Fig. 14. The dependence of the differential photon polarization asymmetry for the $B_s \to \gamma\,\tau^+\tau^-$ decay on the dimensionless variable $x = 2E_\gamma/m_B$ for different values of $C_{\rm LRRL}$ without LD effects

is also the only one which gives the constructive contribution to the SM prediction of H, which we find $H(B_s \to \gamma\,\mu^+\mu^-) = 0.74$. This behavior is also seen from Fig. 10, in which we plot the differential photon polarization asymmetry $H(x)$ for the same decay as a function of x for the different values of the vector interaction with coefficients $C_{\rm RL}$. From these two figures, we can conclude that performing a measurement of H at different photon energies can give information on the signs of the new Wilson coefficients, as well as their magnitudes.

Note that the results presented in this work can easily be applied to the $B_s \to \gamma\,\tau^+\tau^-$ decay. For example, in Figs. 11 and 12, we present the dependence of the $\mathrm{BR}^{(1)}$ and $\mathrm{BR}^{(2)}$ for the $B_s \to \gamma\tau^+\tau^-$ decay on the new Wilson coefficients. We observe that contrary to the $\mu^+\mu^-$ final state, the spectrum of $\mathrm{BR}^{(1)}$ and $\mathrm{BR}^{(2)}$ for the $\tau^+\tau^-$ final state is not symmetrical with respect to the zero point, except for the coefficient $C_{\rm TE}$. Otherwise, we observe three types of behavior for $\mathrm{BR}^{(2)}$ from Fig. 12: as the new Wilson coefficients $C_{\rm LRRL}$, $C_{\rm RLLR}$, $C_{\rm LL}$ and $C_{\rm RR}$ increase, $\mathrm{BR}^{(2)}$ also increases. This behavior is reversed for the coefficients $C_{\rm LRLR}$, $C_{\rm RLRL}$, $C_{\rm LR}$ and $C_{\rm RL}$, i.e., $\mathrm{BR}^{(2)}$ decreases with increasing values of these coefficients. However, the situation is different for the tensor-type interactions: $\mathrm{BR}^{(2)}$ decreases when $C_{\rm T}$ and $C_{\rm TE}$ increase from -4 to 0 and then increases in the positive half of the range. We also observe from Fig. 11 that the spectrum of $\mathrm{BR}^{(1)}$ is identical to that of $\mathrm{BR}^{(2)}$ for the coefficients $C_{\rm LRLR}$, $C_{\rm LRRL}$, $C_{\rm RLLR}$, $C_{\rm LL}$, $C_{\rm RR}$ and $C_{\rm TE}$ in between $-4 \le C_X \le +4$. For the rest of the coefficients, namely $C_{\rm RLRL}$, $C_{\rm LR}$, $C_{\rm T}$, it stands slightly below and almost parallel to the SM prediction in the positive half of the range, although its behavior is the same as $\mathrm{BR}^{(2)}$ when $-4 \le C_X \le 0$.

Finally we present two more figures related to the photon polarization asymmetry H for the $B_s \to \gamma\,\tau^+\tau^-$ decay. Figure 13 shows the dependence of the integrated photon polarization asymmetry H on the new Wilson coefficients. We present the differential photon polarization asymmetry $H(x)$ for the same decay as a function of x for the different values of the scalar interactions with coefficients $C_{\rm LRRL}$ in (14). We see from Fig. 13 that contrary to the $\mu^+\mu^-$ final state, the spectrum of H for the $\tau^+\tau^-$ final state is not symmetrical with respect to the zero point. It also follows that when $0 \le C_X \le 4$ the dominant contribution to H for $B_s \to \gamma\,\tau^+\tau^-$ decay comes from $C_{\rm RLRL}$ and $C_{\rm LR}$. However, for the negative part of the range H

receives constructive contributions mostly from $C_{\rm LRRL}$, as clearly seen also from Fig. 14.

In conclusion, we have studied the total and the differential branching ratios of the rare $B_s \to \gamma \ell^+ \ell^-$ decay by taking into account the polarization effects of the photon. Doing this we use a most general model independent effective Hamiltonian, which contains the scalar- and tensor-type interactions as well as the vector types. We have also investigated the sensitivity of "photon polarization asymmetry" in this radiative decay to the new Wilson coefficients. It has been shown that all these physical observables are very sensitive to the existence of new physics beyond SM and their experimental measurements can give valuable information on it.

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